ARJUNA

JEE

LECTURE NO. - 6

Basic Math

MSM SIR

ARJUNA

TABLE OF CONTENT

01 No-Ne Expressions

02 More about Modulus

MODULUS KA GYAAN

$$|x| = +x; \quad x \geq 0$$

$$= \underbrace{(-x)}_{\geq 0}; \quad x \leq 0$$

$\rightarrow (-x) \geq 0$

ARJUNA

Ulta Soche ⇒

- If $|x| = (+x)$

  Then $x \geq 0$ ★

- If $|x| = (-x)$

  Then $x \leq 0$. ★

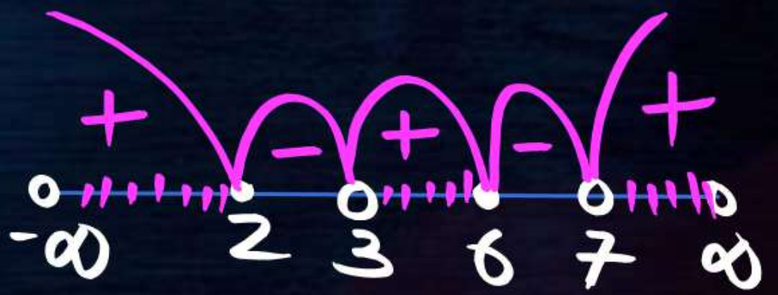
Solve for x:

$$\left|\frac{(x^2-8x+12)}{(-10x+21+x^2)}\right| = \frac{(x^2-8x+12)}{(-10x+21+x^2)}$$

$|E| = (+E) \Rightarrow E \geq 0$

$$\frac{x^2-8x+12}{x^2-10x+21} \geq 0$$

$$\frac{(x-6)(x-2)}{(x-7)(x-3)} \geq 0$$



$x \in (-\infty, 2] \cup (3, 6] \cup (7, \infty)$ Ans.

Solve for x:

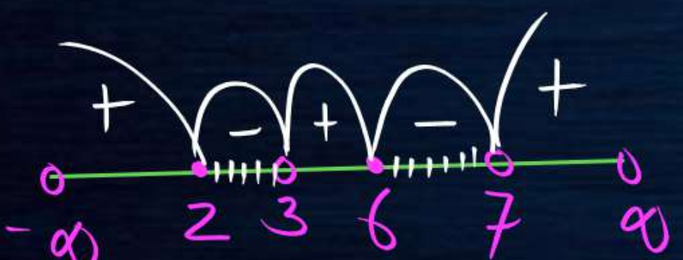
$$\left|\frac{(x^2-8x+12)}{(-10x+21+x^2)}\right| = \frac{-(x^2-8x+12)}{(-10x+21+x^2)}$$

E = $-E$

$|E| = (-E) \Rightarrow E \leq 0$

$$\frac{x^2-8x+12}{x^2-10x+21} \leq 0$$

$$\frac{(x-6)(x-2)}{(x-7)(x-3)} \leq 0$$

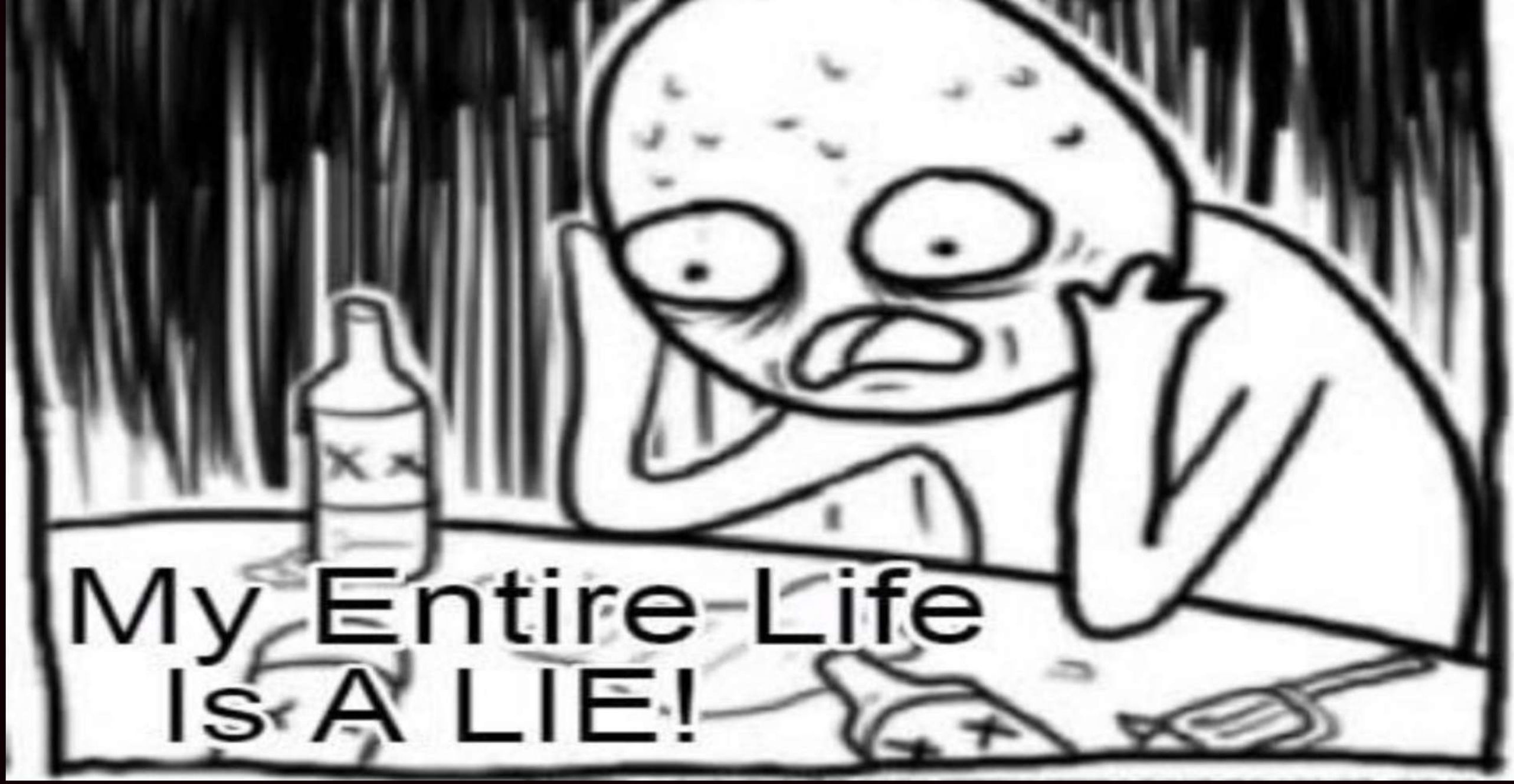


$x \in [2,3) \cup [6,7)$ Ans

SQUARE ROOT KA GYAAN

$$\sqrt{x^2} = |x|$$

ex) $\rightarrow$ $\sqrt{36} = \sqrt{(\pm 6)^2} = |\pm 6| = 6$


My Entire Life
Is A LIE!

Difference b/w $(x^2 = 36)$ and $(\sqrt{36})$

$\sqrt{x^2} = \sqrt{36}$

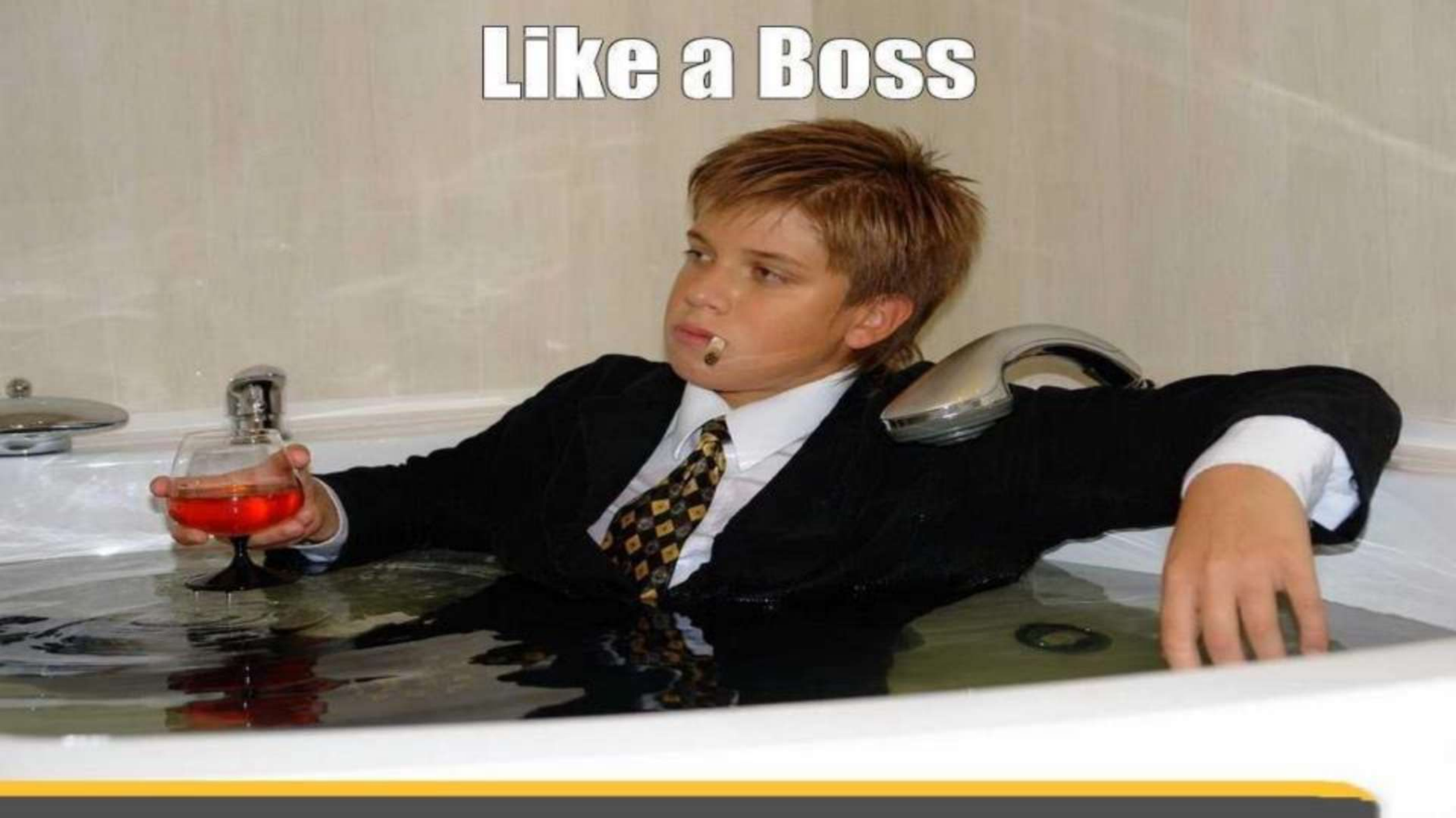
$|x| = 6$

$x = \pm 6$

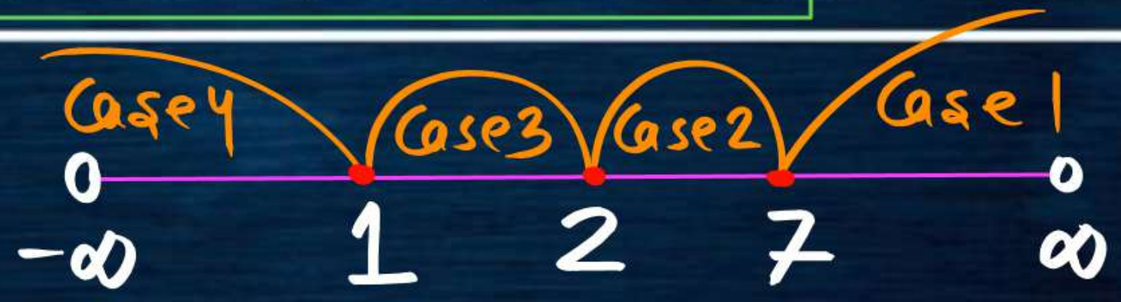
2 values

"6"

only 1 value.


Like a Boss

Q. Simplify $f(x) = |x-1| - 3|2-x| + 5|x-7|$ (HW QUESTION)



Case 1 ⇒ $x \in [7, \infty)$

$$f(x) = (x-1) - 3\{-(2-x)\} + 5(x-7)$$

$$f(x) = (x-1) + 6 - 3x + 5x - 35$$

$$f(x) = 3x - 30$$

Exp. को Represent Karne का tareeka ← func^n in x

Case 2 $\Rightarrow$ $x \in [2, 7)$

$$f(x) = (x-1) - 3\{-(2-x)\} - 5(x-7)$$

$$f(x) = x - 1 + 6 - 3x - 5x + 35$$

$$f(x) = -7x + 40$$

Case 3 $\Rightarrow$ $x \in [1, 2)$

$$f(x) = +(x-1) - 3\{(2-x)\} - 5(x-7)$$

$$f(x) = x - 1 - 6 + 3x - 5x + 35$$

$$f(x) = -x + 28$$

Case 4 $\Rightarrow$ $x \in (-\infty, 1)$

$$f(x) = -(x-1) - 3\{+(2-x)\} - 5(x-7)$$

$$= -x + 1 - 6 + 3x - 5x + 35$$

$$f(x) = -3x + 30$$

final ans →

If $x \in [7, \infty)$, Then $f(x) = 3x - 30$

If $x \in [2, 7)$, Then $f(x) = -7x + 40$

If $x \in [1, 2)$, Then $f(x) = 28 - x$

If $x \in (-\infty, 1)$, Then $f(x) = 30 - 3x$.

Stay Done

TYPE 4

ARJUNA

Q. $\left|\left|3-|x|\right|-5\right| = 13$; Solve for 'x'

$\Rightarrow |E| = 13$

$\Rightarrow E = \pm 13$

$\Rightarrow \left|3-|x|\right| - 5 = \pm 13$

$\left|3-|x|\right| = 5 \pm 13$

(+) $\left|3-|x|\right| = 18$

"q"

$|q| = 18 \Rightarrow q = \pm 18$

(−) $\left|3-|x|\right| = -8$ → "Discard" "CB"

$3-|x|=\pm 18$

$(+)$

$3-|x|=18$

$3-18=|x|$

$|x|=-15$

Discarded

$(-)$

$3-|x|=-18$

$21=|x|$

$x=\pm 21$

OR

$x\in\{-21,21\}$

Ans

$|E_1| = |E_2|$

$\Rightarrow E_1 = \pm E_2$

Solve and "Injoy"

TYPE 5

Q. Solve for x. $|2x-1| = |x-3|$

$E_1 = 2x-1$

$E_2 = x-3$

$$|E_1| = |E_2|$$

$$\Rightarrow E_1 = \pm E_2$$

$(2x-1) = \pm (x-3)$

(+)

$2x-1 = x-3$

$x = -2$

(-)

$2x-1 = -(x-3)$

$2x-1 = -x+3$

$3x = 4$

$x = \frac{4}{3}$

Ans) $x \in \left\{-2, \frac{4}{3}\right\}$

ARJUNA

Q.

Solve for x.

$$|3x^2 - 1| = |x^2 - 10|$$

$x \in R$

?

$E_1 = 3x^2 - 1$; $E_2 = x^2 - 10$

$E_1 = \pm E_2$

$3x^2 - 1 = \pm (x^2 - 10)$

(+)

$3x^2 - 1 = x^2 - 10$

$2x^2 = -9$

$x^2 = -\frac{9}{2}$

Discard

(−)

$3x^2 - 1 = -(x^2 - 10)$

$4x^2 = 11$

$x^2 = \frac{11}{4}$

$$\sqrt{x^2} = \sqrt{\frac{11}{4}}$$

$$\sqrt{x^2} = \frac{\sqrt{11}}{2}$$

$$|x| = \frac{\sqrt{11}}{2}$$

$$\boxed{x = \pm \frac{\sqrt{11}}{2}}$$

Ans

Q. $|3x - 1| = |x + 5|$

$(3x-1) = \pm (x+5)$

(+)

$3x-1 = x+5$

$2x = 6$

$x = 3$

(−)

$3x-1 = -x-5$

$4x = -4$

$x = -1$

$x \in \{-1, 3\}$ Ans

ARJUNA

$|E_1| = (E_2)$

"Zaya Dhyan se"

$E_2 \geq 0$

$E_1 = \pm E_2$

finally check.

TYPE 6

ARJUNA

Q. Solve for x: $|x^2 - x - 6| = x + 2$

$E_1 = x^2 - x - 6$

$E_2 = x + 2$

$E_1 = \pm E_2$

$x^2 - x - 6 = \pm (x + 2)$

(+)

$x^2 - x - 6 = x + 2$

$x^2 - 2x - 8 = 0$

$x = 4, -2$ ✓ ✓

(−)

$x^2 - x - 6 = -x - 2$

$x^2 = 4$

$x = \pm 2$

Check

$x + 2 \geq 0$?

$x = 4$

$x + 2 = 6$ ✓

$x = -2$

$x + 2 = 0$ ✓

$x = 2$

$x + 2 = 4$ ✓

finally,

$x \in \{4, -2, 2\}$ Rule

ARJUNA

Q. Solve for x: $|2x - 5| = x - 1$

$E_1 = 2x - 5$

$E_2 = x - 1$

$x - 1 \geq 0$

$E_1 = \pm E_2$

⊕ $E_1 = E_2$

$2x - 5 = x - 1$

$x = 4$ ✓

⊖ $E_1 = -E_2$

$2x - 5 = -(x - 1)$

$2x - 5 = -x + 1$

$3x = 6$

$x = 2$ ✓

$x = 2$, $x - 1 = 1$ ✓

$x = 4$, $x - 1 = 3$ ✓

Ans $\Rightarrow$ $x \in [2, 4]$

ARJUNA

CASE OF 'No-Ne' FACTORS

"Non-Negative" → Jo negative nhi ho!

→ Positive, zero, ~~Negative~~

Repeated factor $\Rightarrow$ $(x-a)^n$

$n \geq 2$; $n \in N$

- $(x-a)^{\text{Even}}$; ex → $(x-a)^2, (x-a)^4, (x-a)^6, \ldots$

  ↳ No-Ne

- $(x-a)^{\text{odd}}$; ex → $(x-a)^3, (x-a)^5, (x-a)^{201}, \ldots$

  ← Posi°, Negative, zero

$(x-1)^{301}$

$x=2 \rightarrow (2-1)^{301} = 1$ (P) ✓

$x=0 \rightarrow (0-1)^{301} = -1$ (N) ✓

$x=1 \rightarrow (1-1)^{301} = 0$ (Z) ✓

$(x-1)^{\text{Even}}$

$x=2 \rightarrow 1$ (P)

$x=0 \rightarrow (-1)^2$ (P)

$x=1 \rightarrow (0)^2$ (Z)

Summary →

① $(x-a)^{odd}$ → make it $(x-a)$

② $(x-a)^{even}$ → "CB" and "Agni-Parikṣha"

No-Ne factor

"MSM Rule"

Q. Solve the inequality:-

$$(x+5)(2x-3)^5(-x+7)^3(3x+8)^2 < 0$$

CB

"Repeated factor" ?

→ Agnipariksha

$(3x+8)^2 \neq 0$

$3x+8 \neq 0$

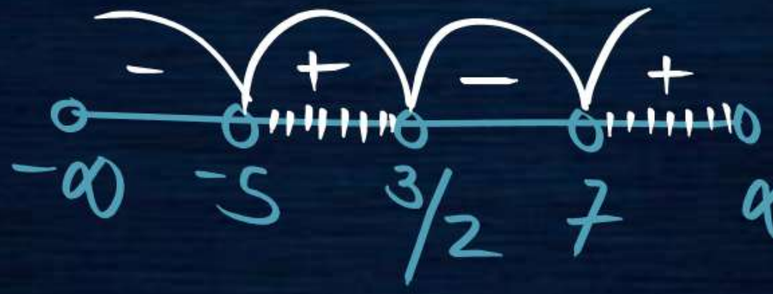
$x \neq -8/3$ ✓

$(x+5)(2x-3)(-x+7) < 0$

$*(-1)$

$-(x+5)(2x-3)(-x+7) > 0$

$(x+5)(2x-3)(x-7) > 0$



$x \in (-5, 3/2) \cup (7, \infty) - \{-8/3\}$ Ans

ARJUNA

Q. (i) $(x-2)^2(x+2)^{16}(x+1) \geq 0$

No-Ne CB

No-Ne CB

Agni-pariksha

$(x-2)^2 = 0$

$x = 2$ ✓

Agni-pariksha

$(x+2)^{16} = 0$

$x = -2$ ✓

$x+1 \geq 0$

$x \in [-1, \infty) \cup \{-2\}$

Ans

?

ARJUNA

Q. (ii) $(x-2)^2(x+2)^{16}(x+1) < 0$

No-Ne
CB
AP ↓
$x \neq 2$ ✓

No-Ne
CB
AP ↓
$x \neq -2$

$x+1 < 0$

$x \in (-\infty, -1) - \{-2\}$

Ans

?

Q. (iii) $(x-1)^3(x+1)(x+2)^6 > 0$

No-Ne
CB
AP

$x \neq -2$

$\Downarrow$

$(x-1)(x+1) > 0$

$$x \in (-\infty, -1) \cup (1, \infty) - \{-2\}$$

Ans

Q. Solving the inequality

$$(x^2 - 3x + 2)(x^3 - 3x^2)(4 - x^2) \geq 0$$

HW

JBK Done

?

HW slide →

① Revise notes (solve)

② DPP का HW ↓

"I'll post"

JBK..JBK...

- TO *FUTURE IIT-IANS*

Thank – You

- FUTURE IIT-IANS